Development of G-Equation Combustion Model for Direct Injection SI Engine Simulations

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Abstract
An improved spark ignition model has been developed and implemented into the KIVA-3V code. In this model, the spark ignition kernel growth is tracked by Lagrangian particle markers. The effects of turbulent flow and spark discharge energy on the kernel growth are considered. After the ignition kernel size exceeds a critical size related to the integral turbulent length scale of the flow, the turbulent flame is considered to be fully developed. A G-equation model is used to track the mean turbulent flame evolution. By ignoring the detailed turbulent flame brush structure, fine numerical resolution is not required, thus making the model suitable for use in multidimensional simulations of SI engines. Combined with a characteristic time scale combustion model, the models were used to simulate the ignition and combustion processes of a small marine DISI two-stroke engine that features two-spark plugs, where the triple flame structure must be considered. The evolution of the two spark kernels and turbulent flames starting from the two spark plugs are modeled. Good agreements with measured cylinder pressure was also obtained.

Introduction
The combustion in spark ignition engines is initiated by an electric discharge from the spark plug. The ignition process can be described by three successive phases — breakdown, arc and glow discharge [Heywood, 1988]. The breakdown phase always precedes the arc and glow discharge. During this phase, an electrically conductive column is created between the spark plug electrodes, where the temperature and pressure is high. Then a shock wave is created that propagates away from the plug. The duration of the initial breakdown phase is very short (~10ns) with high-energy transfer efficiency between the electrical energy supplied and the plasma (about 94%).

During the early stage (0 to 5µs after the breakdown) of the ignition process, the plasma kernel expands violently and the mass and energy transfer processes are much dominated by the pressure wave and the chemical reaction contributes little to the kernel growth. Subsequently, the contribution of chemical reaction becomes significant [Kravchik et al., 1995]. Although the ignition process is important, it is not practical to resolve the process in detail in engine CFD simulations, because the typical grid-size and time step used is larger than that needed to describe this early stage of ignition precisely. Thus, a relatively simple sub-grid scale model is needed to predict the ignition process in 3-D engine calculations.

The combustion in DISI engines can take place in premixed mixture conditions (early injection operating mode) or in partially premixed (stratified) mixture conditions (late injection operating mode). In the late injection mode, by controlling the spray and air-fuel mixing process, a stratified mixture can be obtained with a desirable mixture located near the spark plug. At this operating condition, the equivalence ratio of the stratified air-mixture ranges from rich to lean. The flame propagates with a higher speed along a surface of stoichiometric mixture and with lower speeds on the lean and rich sides. When the unburned intermediate species such as H₂, H, CO diffuse from the rich premixed branch and meet and mix with the O and O₂ surviving from the lean mixture branch, a diffusion flame is developed. Thus, a structure called a “triple flame” is formed, as schematically shown in Fig. 1.

In the present study, this premixed combustion is modeled using the G-equation model, which is a kind of flamelet combustion model, and the diffusion combustion behind, controlled by turbulence mixing, is modeled by a characteristic time scale combustion model. The spray breakup is simulated by LISA [Linearized Instability Sheet Atomization] model [Schmidt et al, 1999]. The ignition and combustion models are implemented into the KIVA-3V code, which is used in this study.

Ignition Model Development
In the present work, the ignition model used the same approach as that used in the DPIK model of Fan et al. [1999]. The kernel surface position is marked by the particles, and the flame surface density is obtained from the number density of particles in each computational cell, as shown in Fig. 2.

The ignition kernel is assumed to be spherical during the initial ignition process. When the kernel grows, the particles move outwards radially from the spark plug electrodes. The ignition particle’s speed, i.e., the kernel growth rate, is influenced by the flow turbulence and air-
fuel mixture stoichiometry. When the kernel becomes large, different ignition kernel particles occupy different computational cells, where the flow turbulent intensity and air-fuel mixture stoichiometry may be different. Thus, the ignition kernel can grow at different speeds in different directions. Thus, the shape of the ignition kernel is not spherical any more.

\[
\dot{m}_k = \frac{dm_k}{dt} = \rho_u A_k s_{\text{eff}} = \rho_u A_k (s_i + s_{\text{plasma}}) \quad (1)
\]

where \( \rho_u \) is unburned gas density, \( A_k \) is the flame kernel surface area, and \( s_{\text{eff}} \) is the effective kernel growth speed. The latter is the sum of turbulent flame speed \( s_t \) and the so-called plasma velocity \( s_{\text{plasma}} \), which results from the spark discharge energy.

In addition, using \( m_i = \rho_i V_i \), the mass-burning rate can be rewritten as

\[
\frac{dm_k}{dt} = \frac{d(\rho_i V_i)}{dt} = \rho_i \frac{dV_i}{dt} + V_i \frac{d\rho_i}{dt} = \rho_i A_i (s_i + s_{\text{plasma}}) \quad (2)
\]

where \( \rho_i \) and \( V_i \) are gas density and kernel volume respectively, and subscript \( k \) indicates the kernel.

Utilizing the ideal gas law,

\[
\frac{1}{\rho_k} \frac{d\rho_k}{dt} = \frac{1}{p} \frac{dp}{dt} - \frac{1}{T_k} \frac{dT_k}{dt} \quad (3)
\]

the following equation is obtained:

\[
\frac{dV_k}{dt} = \frac{\rho_u}{\rho_k} A_k (s_i + s_{\text{plasma}}) + V_k \left( \frac{1}{T_k} \frac{dT_k}{dt} - \frac{1}{p} \frac{dp}{dt} \right) \quad (4)
\]

Normalizing the volume increase to the flame kernel surface area \( A_k \), the mean expansion velocity of the flame kernel can be obtained as

\[
\frac{dr_k}{dt} = \frac{\rho_u (s_i + s_{\text{plasma}})}{\rho_k} + \frac{V_k}{A_k} \left( \frac{1}{T_k} \frac{dT_k}{dt} - \frac{1}{p} \frac{dp}{dt} \right) \quad (5)
\]

During the ignition process, the pressure increase due to the combustion is negligible. We also assume the temperature inside the kernel is spatially uniform and equal to the adiabatic flame temperature of the air-fuel mixture. This assumption is feasible because the breakdown phase, which is characterized by high temperatures within the plasma channel, lasts only few nanoseconds. The majority of the spark energy is transferred during arc and glow discharge phases. The arc and glow discharge phases last longer (of the order of milliseconds) and the temperature within the kernel does not differ much from the adiabatic flame temperature.

With these two assumptions, the kernel growth rate is simplified as

\[
\frac{dr_k}{dt} = \frac{\rho_u (s_i + s_{\text{plasma}})}{\rho_k} \quad (6)
\]

This equation is used to calculate the movement of the kernel particles. The plasma velocity \( s_{\text{plasma}} \) can be obtained from the energy balance equation as follows.

During the ignition process, the energy balance for the control volume surrounded by ignition kernel surface is given by

\[
\dot{Q} = \left( \frac{dE}{dt} \right)_{CV} - (h_u + \frac{1}{2} \nu_{\text{unburned}}^2) \dot{m}_{\text{unburned}} + p \frac{dV}{dt}_{CV} \quad (7)
\]

where \( \dot{Q} \) is the net heat transfer rate to the kernel, which is the sum of the electrical energy transfer rate to the kernel \( \dot{Q}_{\text{el}} \), the heat loss rate to the electrodes \( \dot{Q}_{\text{el}} \), and the reaction heat release rate \( \dot{Q}_{\text{chem}} \). Therefore

\[
\dot{Q} = \dot{Q}_{\text{el}} + \dot{Q}_{\text{chem}} - \dot{Q}_{\text{el}} \quad (7)
\]

The \( V_{\text{unburned}} \) is the velocity of unburned gas which is on the flame kernel surface. Compared with the unburned gas enthalpy, this unburned gas velocity can be neglected. Thus, Eq. (7) can be rewritten as
\[
\dot{Q}_{\text{spk}} + \dot{Q}_{\text{chem}} - \dot{Q}_{\text{hl}} = \left( \frac{dE}{dt} \right)_{\text{CV}} - h_m \dot{m}_k + p \left( \frac{dV}{dt} \right)_{\text{CV}} \quad (8)
\]

The change in flame kernel internal energy \(\left( \frac{dE}{dt} \right)_{\text{CV}}\) is

\[
\left( \frac{dE}{dt} \right)_{\text{CV}} = m_k \frac{dU_k}{dt} + U_k \frac{dm_k}{dt} = m_k \frac{dU_k}{dt} + U_k \dot{m}_k \quad (9)
\]

where \(U_k\) is the specific internal energy of the burned gas within the flame kernel.

If the ignition kernel shape is spherical, the change in volume work is

\[
p \left( \frac{dV}{dt} \right)_{\text{CV}} = 4\pi \rho_k^2 \frac{dr_k}{dt} = 4\pi \rho_k^2 \frac{P_k}{\rho_k} (s_t + s_{\text{plasma}}) \quad (10).
\]

Combining Eqs. (9) and (10), Eq. (8) is rewritten as

\[
\dot{Q}_{\text{spk}} + \dot{Q}_{\text{chem}} - \dot{Q}_{\text{hl}} = m_k \frac{dU_k}{dt} + U_k \dot{m}_k - h_m \dot{m}_k + 4\pi \rho_k^2 \frac{P_k}{\rho_k} (s_t + s_{\text{plasma}}) \quad (11)
\]

If no spark discharge energy is put into the kernel and no heat is lost to the electrodes, the energy conservation equation (11) becomes

\[
\dot{Q}_{\text{chem}} = m_k \frac{dU_k}{dt} + U_k \dot{m}_k = m_k \frac{dU_k}{dt} + U_k \dot{m}_k - h_m \dot{m}_k + 4\pi \rho_k^2 \frac{P_k}{\rho_k} (s_t + s_{\text{plasma}}) \quad (12)
\]

where \(U_{\text{ad}}\) is the burned gas specific internal energy at adiabatic flame temperature.

The mass-burning rate is only related to the turbulent flame velocity, and can be written as

\[
\dot{m}_{\text{flame}} = \rho_s A_s s_t \quad (13)
\]

Subtracting Eq. (12) from Eq. (11), the equation

\[
\dot{Q}_{\text{spk}} - \dot{Q}_{\text{hl}} = m_k \left( \frac{dU_k}{dt} - \frac{dU_k}{dt} \right) + U_k \dot{m}_k - U_{\text{ad}} \dot{m}_{\text{flame}} + 4\pi \rho_k^2 \frac{P_k}{\rho_k} (s_t + s_{\text{plasma}}) \quad (14)
\]

is obtained.

According to the assumption that the gas temperature within the kernel equals the adiabatic flame temperature, \(U_{\text{ad}} = U_k\). And also using

\[
\dot{m}_k - \dot{m}_{\text{flame}} = 4\pi \rho_k^2 \rho_s \dot{s}_{\text{plasma}} \quad (13)
\]

Eq. (14) is written as

\[
\dot{Q}_{\text{spk}} - \dot{Q}_{\text{hl}} = 4\pi \rho_k^2 \rho_s (U_k - h_u) \dot{s}_{\text{plasma}} + 4\pi \rho_k^2 \frac{P_k}{\rho_k} \dot{s}_{\text{plasma}} \quad (15)
\]

\[
\dot{Q}_{\text{spk}} - \dot{Q}_{\text{hl}} = \dot{Q}_{\text{spk}} \cdot \eta_{\text{eff}} \quad (16)
\]

where \(\eta_{\text{eff}}\) is electrical energy transfer efficiency, which is affected by the plug geometry. Most of the ignition process occurs in the glow discharge phase and \(\eta_{\text{eff}}\) is relatively low (\(\approx 30\%\)) [Heywood, 1988].

Finally, the plasma velocity \(s_{\text{plasma}}\) is obtained as

\[
s_{\text{plasma}} = \frac{\dot{Q}_{\text{total}} \cdot \eta_{\text{eff}}}{4\pi \rho_k^2 \left( \frac{P_k}{\rho_k} (U_k - h_u) + p \frac{\rho_k u}{\rho_k} \right)} \quad (17)
\]

During the ignition kernel formation and initial development process, the turbulence integral length scale is typically much larger than the kernel size so that the kernel can only sense the high frequency fraction of the turbulence spectrum. The influence of kernel size on the turbulence flame velocity, \(s_t\) is given by Herweg et al. [1992]:

\[
\frac{s_t}{s_j} = I_0 + I_0^{1/2} \left( \frac{u'}{u' + s_t} \right)^{1/2} \left( 1 - \exp \left( - \frac{r_k}{l_s} \right) \right)^{1/2} \quad (18)
\]

where \(u'\) is the turbulence intensity, \(s_j\) is the laminar burning velocity, \(l_s\) is the turbulence integral length scale (see Eq. 24). \(T_{\text{eff}}\) is a characteristic time scale:

\[
T_{\text{eff}} = \frac{l_s}{u' + s_t} \quad (19)
\]

To account for the effects of strain on the kernel development, \(I_0\) is obtained from

\[
I_0 = 1 - \left( \frac{l_s}{l_f} \right)^{1/2} \left( \frac{u'}{s_j} \right)^{3/2} - 2 \frac{l_s}{r_k} \frac{P_0}{P_s} \quad (20)
\]

where \(l_f\) is thickness of laminar flame (see Eq. 29). The laminar speed \(s_j\) is estimated by using the correlation of Metghalchi and Keck [1980].

\[
s_j = \left( s_1 - s_2 \left( \phi - s_3 \right)^2 \right) \left( \frac{T}{298} \right)^{\alpha_1 - \alpha_2 (\phi - 1)} \times \left( \frac{P_k}{1013} \right)^{\beta_1 + \beta_2 (\phi - 1)} \left( 1 - 0.1 R \right) \quad (21)
\]

where \(\phi\) is equivalence ratio, \(R\) is the residual gas fraction, \(P_0\) is the burnt gas pressure and \(T\) is the estimated local unburned gas temperature. The other parameters are model constants, which can be found in Metghalchi and Keck [1980].

The plasma velocity is inversely proportional to \(r_k^2\), and thus decreases rapidly as the ignition kernel grows. It starts out from a very large value because of the plasma effect, and then its influence decreases rapidly as the kernel become large, which is schematically displayed in Figure 4. It also can be seen that the turbulent flame speed, \(s_t\), increases with time, while the laminar flame speed \(s_j\) does not change much with time. The \(s_t\) curve indicates that the range of turbulence frequencies sensed by the kernel flame becomes enlarged as the kernel grows.

In the derivation of Eq. (17), we assumed the shape of flame kernel is spherical. When the kernel becomes large, its shape may be changed because the turbulent flame speed \(s_t\) for different kernel particles may be different. But Eq. (17) can still be applied in this situation. The kernel is assumed to grow starting from a sphere with radius of 0.5 mm. Only when the kernel size is bigger than the computational mesh size (usually 2-4mm), can
the kernel change to other shapes. At this time, the speed \( s_{\text{plasma}} \) is very small compared to the turbulent flame speed \( s_t \). In this case, the plasmas effect is not important any more.

\[
\begin{align*}
\text{Figure 4 History of plasma velocity and turbulent flame speed in a premixed SI engine with engine speed 1600 rev/min and ignition timing –30 ATDC}
\end{align*}
\]

The species density changes are calculated by

\[
\frac{d \rho_i}{dt} = C_w \rho_u (Y_i^u - Y_i^b) \sum s_i
\]

(22)

where \( \sum \) is the flame surface density within a particular cell,

\[
\sum = \frac{N_{p, cell}}{N_{p, tot}} \frac{4 \pi r_i^2}{V_{cell}}
\]

(23)

Here \( N_{p, cell} \) is the number of ignition particles in the computational cell, \( V_{cell} \) is the cell volume, \( N_{p, tot} \) is the total number of ignition particles and \( C_w \) is a constant chosen equal to 10 to assure complete combustion inside the ignition kernel. \( Y_i^u \) and \( Y_i^b \) are the mass fractions of species, \( i \), in the unburned and burnt gas, respectively, and \( \rho_u \) is the unburned gas density. Seven species, fuel, \( \text{O}_2 \), \( \text{H}_2\text{O}, \text{CO}_2, \text{CO}, \text{N}_2 \) and \( \text{H}_2 \) are considered in the calculations. \( Y_i^b \) is assumed to be the local and instantaneous thermodynamic equilibrium value of the mass fraction of species \( i \).

Once the ignition kernel exceeds a critical radius that is related to the integral turbulent length scale, \( l \), of the flow field, i.e.,

\[
r_{12} \geq C_{\text{ot}} l = C_{\text{ot}} \cdot 0.16 \left( \frac{k^{1.5}}{\varepsilon} \right) \]

(24)

the ignition model switches to the combustion model. Here \( k \) and \( \varepsilon \) is the turbulence kinetic energy and its dissipation rate, respectively, and \( C_{\text{ot}} \) is a model constant set equal to 1.2.

**Combustion Model Development**

In spark ignition engines, the premixed flame, where the chemical reactions take place, is relatively thin. This thin flame separates the unburned and burnt gas. In spark ignition engines, the turbulent eddies can wrinkle and strain the flame surface, but the inner flame layer, where the fuel is consumed and radicals are depleted by chain-breaking reactions, is not perturbed [Seshadri and Peters, 1990]. The turbulent flame can be viewed as ensemble of thin reaction-diffusion layers, called flamlels, embedded within a non-reacting turbulent flow. The advantage of the flamelet concept is that the calculation of chemical reaction and turbulent flow can be decoupled.

To track the premixed flame front evolution, a field equation of the scalar \( G(x,t) \) is used, where the flame front is represented by level surface \( G(x,t) = 0 \). This interface divides the flow field into an unburned region, \( G<0.0 \), and a burnt gas region, \( G>0.0 \) [Williams, 1985, Peters, 2000]. In KIVA-3V calculations, the positions of vertices where the velocities are defined and able to move to facilitate the application of moving boundary conditions (such as the piston and the valves). Considering this feature, the equation for the mean location of the turbulent flame front can be written as

\[
\frac{\partial G}{\partial t} + (\mathbf{v}_f - \mathbf{v}_{\text{vert}}) \cdot \nabla G = \frac{\mathbf{F}_x}{\rho G} \nabla \cdot \nabla G - D \nabla^2 G
\]

(25)

Here \( \mathbf{G} \) is the Favre mean value of \( G \), and the mean location of the turbulent flame front is defined by \( \mathbf{G}(x,t) = 0 \). \( \rho \) is the unburned gas density and \( \mathbf{G} \) is the gas density at the mean location of the turbulent flame. \( k_M \) is the mean flame curvature that can be expressed in terms of the level set function \( \mathbf{G} \) as

\[
k_M = \nabla \cdot \left( \frac{\nabla G}{|\nabla G|} \right)
\]

(26)

\( \mathbf{v}_f \) is the Favre mean bulk flow velocity at the flame front and \( \mathbf{v}_{\text{vert}} \) is the velocity of moving vertices. \( D \) is the turbulent diffusivity. The turbulent burning velocity can be can be evaluated from [Peters, 1999]

\[
s_t = s_t + v' \left\{ -a_4 b_1^2 \frac{D a}{a_4 b_1 b_2} \left( \frac{a_4 b_2^2}{2 b_1} \right)^2 \right\}^{1/2}
\]

(27)

where \( v' \) is the turbulence intensity, equal to \( (2k/3)^{1/2} \) and \( a_4 = 0.78, b_1 = 2.0 \) and \( b_3 = 1.0 \) are constants derived from the turbulence model.

\[
Da = \frac{s_t^4 l}{v' \ell_f}
\]

(28)

where \( l \) is the turbulence integral length scale and

\[
\ell_f = \left( \frac{\lambda}{c_p} \frac{\rho_{s_t}}{u} \right)
\]

(29)

is the flame thickness. The heat conductivity, \( \lambda \), and heat capacity, \( c_p \), are evaluated at the inner layer temperature.
The laminar burning velocity, \( s_0 \), and the density is evaluated in the unburnt gas, as shown in Eq. (21).

In a typical multidimensional computational mesh, the average length of the computational cells is usually 2 to 4 mm (if the size is reduced further, the computations become impractically expensive). In some engines, the flame brush thickness is of the same order as the cell size. Thus, the detailed turbulent flame brush structure is hard to capture using a relatively coarse mesh.

A new method is proposed here to calculate the species density change in the cells containing the mean flame front, viz.,

\[
\frac{d \rho_i}{dt} = \frac{\rho_i (Y_i^u - Y_i^b)}{V(i4)} \frac{\text{area}_i(i4)}{s^0_i} \tag{30}
\]

where \( Y_i^u \) and \( Y_i^b \) are the mass fractions of species, \( i \), in the unburned and burnt gas, respectively, and \( \rho_u \) is the unburned gas density. \( \text{area}_i(i4) \) is the mean flame surface area in a cell \( i4 \). Here \( i4 \) is the KIVA-3V cell index number [Amsden, 1997]. As in the ignition model, seven species, fuel, \( \text{O}_2 \), \( \text{H}_2\text{O}, \text{CO}_2 \), \( \text{CO} \), \( \text{N}_2 \) and \( \text{H}_2 \) are considered in the calculations. \( Y_i^u \) is assumed to be the local and instantaneous thermodynamic equilibrium value of the mass fraction of species \( i \).

The numerical scheme to solve the \( \bar{G} \) equation includes initialization, \( \bar{G} \) value update, and reinitialization. The narrow band solution methodology is also employed in the model [Tan and Reitz, 2003a]. In this method, the mean flame surface area is calculated using the piercing points of the flame front with the cell edges [Tan and Reitz, 2003a].

Behind the premixed flame branches, a diffusion flame develops to consume the excess fuel and oxidizer surviving from the rich and lean premixed flame branches separately. A laminar and turbulent characteristic time scale combustion model [Abraham, 1985] is applied to calculate the conversion rate of each species behind the premixed flame branches. The conversion rate of each species is assumed to be

\[
\frac{d \rho_i}{dt} = - \left( \frac{\rho_i - \rho_i^*}{\tau_c} \right) \tag{31}
\]

where \( \rho_i \) is the density of species \( i \), and \( \rho_i^* \) is the local and instantaneous thermodynamic equilibrium value of the density of species \( i \). \( \tau_c \) is the characteristic time to achieve such equilibrium and it is assumed to be the same for the 7 different species. Because the temperature of the gas behind the premixed flame branches is already high, the species conversion rate is controlled by the turbulent mixing time \( \tau_m \), only, and \( \tau_c \) can be assumed to be equal to \( \tau_m \), that is

\[
\tau_c = \tau_m = c_{m2} \frac{k}{\varepsilon} \tag{32}
\]

where \( c_{m2} = 0.018 \).

Results and Discussion

A small DISI two-stroke engine, which is used widely in marine applications, was investigated since experimental data is available [Hudak, 1998]. The engine has a flat-top piston and a Chrysler-designed high-pressure swirl-type injector that is centrally mounted in the dome-shaped cylinder head.

The engine specifications and operating conditions are listed in Table 1. The computing mesh is shown in Fig. 5. It is comprised of about 11,000 computational cells. Two spark plugs are located on the boost port and exhaust port sides separately (see Fig. 7). The plugs reach into the dome is about 12 mm. Each spark plug is represented by about 2,200 particles. Due to its existence, the gas velocity near the plug is reduced [Fan, et al. 1999.]

| Bore (mm) | 85.8 |
| Stroke (mm) | 67.3 |
| True Compression Ratio | 7.4 |
| Exhaust Port Timing (°ATDC) | 95 |
| Boost Port Timing (°ATDC) | 117 |
| Transfer Port Timing (°ATDC) | 117 |
| Speed (rpm) | 2000 |
| Injection Timing (°ATDC) | 280 |
| Ignition Timing (°ATDC) | 334 |

Table 1 Engine Specification and Operating Conditions

Figure 7 shows the growth of ignition kernels and the flame front evolution. As discussed above, the ignition kernel is represented by marker particles. The flame front, defined by \( G(\delta,t) = 0 \), is represented by light (yellow) surface in this figure.

Inside the ignition kernel, the fuel is burned and the gas temperature is increased. Since the air-mixture stoichiometry on the exhaust port side (right hand side in Fig. 6) is more suitable to burn, the ignition kernel on this side grows faster than the left ignition kernel, as shown in Fig. 7. When the radius of ignition kernels becomes larger than a critical size, the G-equation combustion model starts to work.

Figure 6 shows the air-fuel ratio in the engine symmetry plane at crank angle 335°.
simulation, the model constant $C_{nl}$ in Eq. 24 is set to 1.2. As also can be seen in Fig. 7, after the right hand side ignition kernel switched to the combustion model, the left side ignition kernel is still growing. The two separate flame fronts merge to one flame after they meet.

Figure 7 Evolution of the ignition kernels and flame front (defined by $G(x,t) = 0$ represented by the light (yellow) surface).

The predicted cylinder pressure trace is shown in Fig. 8. The agreement between the predicted and measured results is satisfying.

These models have also been applied to the premixed combustion simulations and stratified combustion simulations in another DISI engine. Those results are also very encouraging [Tan and Reitz, 2003a, Tan and Reitz, 2003b].

**Conclusions**

An equation to calculate ignition kernel growth rate is derived in this study. The kernel growth rate is highly influenced by the spark discharge energy in the early stage of the ignition process. After the kernel becomes bigger, the effects of flow turbulence on the kernel growth rate become apparent. To make the model suitable to 3-D engine simulations, the kernel flame surface positions are marked by Lagrangian Marker particles. Even when the ignition kernel size is smaller than the computational grid size, the kernel growth can still be tracked well. Thus the use of very fine numerical mesh to predict the ignition process is not needed.

The evolution of fully developed premixed turbulent flames is tracked using the level set method (G-equation). The diffusion combustion behind the premixed flame branches is modeled using a modified characteristic time scale model. In practical engine calculations, the size of the computational cells is of the same order as the thickness of the turbulent flame brush. It is shown that the detailed turbulent flame brush structure can be ignored in the present study. The model was applied to predict the combustion process in a DISI engine and the results agree well with available experimental cylinder pressure data.

**Acknowledgement**

The authors acknowledge the financial support of Ford Motor Company.

**Reference**