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A Turbulence Dissipation Correction to the k-epsilon Model and Its Effect on Turbulence Length Scales in Engine Flows

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Abstract

Non-equilibrium turbulence considerations from rapid distortion theory have been utilized to derive a correction to the turbulence dissipation rate, which then has been implemented in combination with the RNG k-epsilon model in a KIVA-based code. This correction reflects the time delay between changes in the turbulent kinetic energy due to changes in the mean flow and its turbulence dissipation rate, and it is shown that this time delay is controlled by the turbulence Reynolds number. This dissipation rate correction has been validated with experimental turbulence measurements of a small diesel engine operated in motored mode. Improvements in the turbulence length scales have then been obtained by means of this model correction for the compression and the combustion phase of two heavy-duty DI diesel engines.

Introduction

The linear two-equation k-ε model is a widely used turbulence model based on the eddy-viscosity approach, where the turbulent kinetic energy, k, and the turbulent dissipation, ε, are described by transport equations [9, 10, 12]. This model has been successfully tested in a wide variety of steady state flows occurring in technical applications. However, one of the model's shortcomings is in transient flows where the equilibrium turbulence hypothesis is violated. In fact, due to piston movement, the fuel injection and the combustion process, the flows in DI diesel engines are transient in nature and therefore, the equilibrium assumption of turbulence is not satisfied [17].

Various approaches have been investigated by numerous researchers in order to remedy the shortcomings of the linear k-ε-based turbulence models. Improvements of this model, based on the theory of re-normalization groups (RNG), have been achieved by Yakhot et al. [21] and successfully introduced to spray combustion simulations by Han et al. [8]. However, uncertainties in the dissipation transport equation still remain and have been investigated in a recent study by Bianchi et al. [2]. In this investigation, the change of the molecular viscosity, predicted by the rapid distortion theory, have been integrated into the ε-equation. This is a further development of a study by Coleman and Mansour [3], where an additional closure relation has been developed to account for rapid distortion effects.

In the present study, a correction for the turbulence dissipation, ε, based on non-equilibrium turbulence considerations from the rapid distortion theory, has been implemented in the *RNG* $k-\varepsilon$ model in a *KIVA*-based code. The model correction has been validated with experimental values of the turbulence integral length scale of a small experimental *Lombardini* engine in motored operating mode. Improvements to the turbulence predictions, utilizing this dissipation correction, are then discussed in comparison with the standard *RNG* $k-\varepsilon$ model in the compression and the combustion phase of the two heavy-duty DI diesel engines, the *Caterpillar 3406* (CAT) and the *Sulzer S20* (S20). The main engine specifications are summarized in Table 1.

Turbulence Considerations

The equilibrium hypothesis, which states that the instantaneous turbulence production rate equals the turbulence dissipation rate, is one of the fundamental assumptions in eddy viscosity-based turbulence modeling. If only one length scale is used to characterize turbulence, say the macro length scale L_ε as is the case in the 2-equation k-ε model, then L_ε represents all the length scales of the turbulence, and the turbulence can be described as being in equilibrium. In this case, the turbulence is self-similar since all scales adjust to flow changes at the same rate.

Based on the equilibrium hypothesis, the dissipation rate, ε_{eq}, can be expressed as

$$\varepsilon_{eq} \propto q^3/L_I$$

where $q = \sqrt{2k/3}$ is the turbulence intensity, k the turbulence kinetic energy and L_I denotes the integral length scale. For equilibrium turbulence it is assumed that $L_I \propto$

Table 1: Specifications for the simulated engines.

	Lombardini	*Caterpillar 3406*	*Sulzer S20*
Bore [mm] x stroke [mm]	86 x 75	137 x 165	200 x 300
Engine speed [rev/min]	1500 (motored)	1600	1000
Inlet valve closure [deg TDC]	-100	-147	-144
Orifices x diameter [mm]	–	6 x 0.259	12 x 0.285
Swirl ratio	–	0.67	–
Injection system	–	common rail	conventional
Injection pressure (avg.) [MPa]	–	90	95
Cells at TDC: rad. $\times$ azym. $\times$ height	30x51x14	24x30x14	23x13x14

L_ε which yields the relation

$$\varepsilon_{eq} = C_\varepsilon k^{3/2}/L_\varepsilon \tag{1}$$

where C_ε is a constant.

For highly transient flows, however, the equilibrium turbulence assumption is likely to be violated, and Eq. (1) does not hold. In fact, the energy dissipation cascade adjusts to changes in the mean flow with some delay and different length scales adjust at different rates. Therefore, a non-equilibrium formulation for the dissipation is required. It is shown in [19, p.67], that for isotropic, homogeneous turbulence, the dissipation can be expressed by the Taylor relation

$$\varepsilon = 10\nu_o k/L_\lambda^2 \tag{2}$$

where ν_o is the molecular (kinematic) viscosity and L_λ denotes the Taylor micro scale.

Wu et al. [20] performed DNS computations for flows under rapid compression conditions. They found that the Taylor length scale, L_λ, and the integral length scale, L_I, remain proportional to each other during a rapid compression. A similar finding has been made experimentally by Dinsdale et al. [4] in engine measurements. The result is also consistent with that obtained from a rapid distortion analysis of isotropic compressed turbulence given by Reynolds [14] and Wu et al. [20]. Using the fact that $L_I \propto L_\lambda$, Eq. (2) becomes

$$\varepsilon \propto \nu_o k/L_I^2 \tag{3}$$

From equations (1) and (2) Han et al. [7] derived the relation

$$L_\varepsilon \propto Re_t L_I \tag{4}$$

where $Re_t = qL_I/\nu_o$ is the turbulent Reynolds number.

Comparison of formulae (1) and (3) yields the following relation between the equilibrium and non-equilibrium dissipation rates

$$\varepsilon \propto \varepsilon_{eq}/Re_t \tag{5}$$

Eq. (5) is the key in the interpretation of the time delay between a change in the turbulence kinetic energy, k, and its dissipation rate ε. A change in the mean flow results in an immediate change in k, whereas the turbulence dissipation, which takes place on the smallest length scale, occurs with some delay. In fact, this delay lies in the order of one eddy turn-over time given by $\tau_\varepsilon = L_I/q$. The relation between this time delay and the turbulence Reynolds number is given by the time scale ratio

$$Re_t = qL_I/\nu_o = \tau_m/\tau_\varepsilon \tag{6}$$

where $\tau_m = L_I^2/\nu_o$ is the molecular diffusion time scale, which is independent of the turbulence behavior. According to Eq. (6), a change in the eddy turn-over time τ_ε results directly in a change of Re_t and consequently, Eq. (5) is the statement that the delay in the dissipation rate is controlled by the turbulence Reynolds number.

It follows from dimensional analysis that the turbulent viscosity, ν_t, can be expressed as

$$\nu_t \propto k^2/\varepsilon \tag{7}$$

The turbulent Reynolds number, $Re_t = qL_I/\nu_o$, depends on the integral length scale L_I and can be expressed in terms of the local variables via Eq. (3) as $L_I \propto \sqrt{\nu_o k/\varepsilon}$. This leads to $Re_t \propto \nu_o^{-1/2} k/\sqrt{\varepsilon}$ and, after identifying $\sqrt{\nu_t} \propto k/\sqrt{\varepsilon}$, (using relation (7)), one obtains

$$Re_t \propto \sqrt{\nu_t/\nu_o}. \tag{8}$$

Note that this expression is different from the one obtained under the assumption of equilibrium turbulence, namely, $Re_t \propto \nu_t/\nu_o$.

Models

The computations presented in this study have been performed with an enhanced version of the *KIVA* code [1]. This code is equipped with many new or improved models including the wave atomization and drop breakup model as originally developed for Kelvin-Helmholtz instabilities by Reitz [13], with further modifications incorporating the Rayleigh-Taylor instability as reported in Su et al. [16] and Ricart et al. [15]. Additional improvements include the wall heat transfer model of Han and Reitz [6], the Shell ignition model of Halstead et al. [5] as implemented by Kong et al. [11], and the characteristic time combustion model developed by Kong et al. [11]. For the low temperature chemistry ($T < 1000\ K$) the chemical reactions

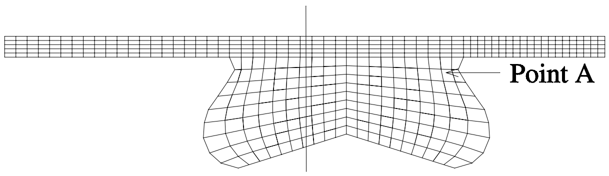

Figure 1: Cross-section at TDC of the experimental *Lombardini* engine showing the observation point A 5mm below the cylinder head and 20mm from the cylinder axis. The 6mm piston bowl off-set is indicated by the cylinder axis (vertical line).

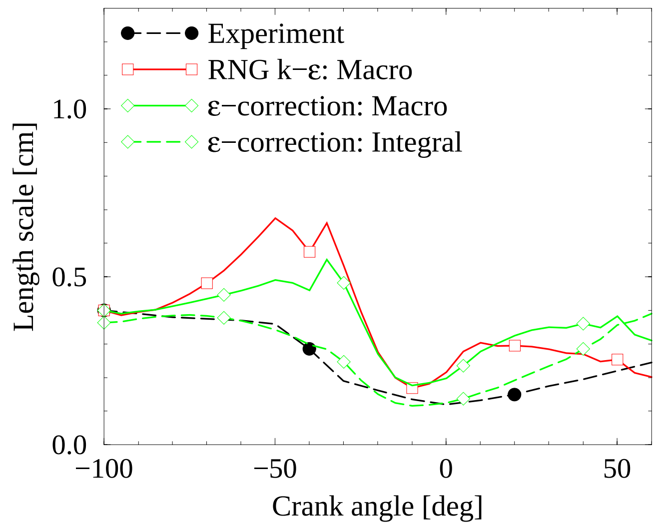

Figure 2: Measured and computed integral and macro length scales at the observation point A.

are described with the Shell ignition model, whereas for the medium and high temperature range ($T \geq 1000\ K$) the characteristic time combustion model is applied.

The employed turbulence model is the *RNG* $k - \varepsilon$ turbulence model as implemented by Han and Reitz [8], adapted to account for the non-equilibrium turbulence effects discussed in the previous section.

Non-Equilibrium Turbulence Modeling

A rapid change in the fluid flow leads to an immediate change of the turbulence production (via the mean flow) on the integral end of the turbulence spectrum. The turbulence dissipation, which occurs on the other end of the spectrum, is adjusting to this change with some delay. As discussed in the previous section, a delay in the turbulence dissipation rate is controlled by the change of the turbulence Reynolds number, Re_t. Therefore, an adjustment of the turbulence dissipation is required in each iteration step. This correction is performed according to Eq. (5) where Re_t is computed via Eq. (8) and the constant of proportionality is taken to be the turbulence Reynolds number from the previous time step.

If there is no change in the flow, then there is no change in the turbulent Reynolds number and, consequently, $\varepsilon = \varepsilon_{eq}$. In this situation the model reduces to the usual *RNG* $k - \varepsilon$ model.

Results and Discussion

The objectives of this study are the validation and implementation of a correction to the turbulence dissipation rate which is based on the non-equilibrium turbulence effects discussed earlier. The dissipation correction has been validated with experimental data from a modified single-cylinder *Lombardini* diesel engine operating in motored mode at 1500 rpm. Compression and combustion computations have been performed for the CAT and the S20 engines using the standard *RNG* $k - \varepsilon$ model and the ε-correction. The focus of these investigations has been on the turbulence integral and macro length scales.

For the simulation of the non-reacting phases, all the model parameters have been kept the same for all computations, with or without the ε-correction. The combustion simulations, however, required an adjustment of the ignition delay and an adjustment of the turbulence characteristic time coefficient C_M. With the *RNG* $k - \varepsilon$ model, C_M has been set to $C_M = 0.25$ for the CAT and $C_M = 2.5$ for the S20 in order to match the experimental cylinder peak pressures (c.f. [18]). For the computations with the dissipation correction, the value $C_M = 2.0$ has been found to yield good agreement with the experimental cylinder pressure data of both engines.

Further, it should be remarked that the quantities represented in the diagrams are mass-averaged over the entire combustion chamber, denoted by $< \cdots >$, at the respective crank angle.

Determination of the Integral Length Scale

For equilibrium turbulence the length scale satisfies $L_I \propto L_\varepsilon$, whereas for non-equilibrium turbulence $L_I \propto L_\lambda$. In simulations, L_ε and L_λ are computed from Eq. (1) and Eq. (2), respectively, and therefore, L_I is determined only up to a constant of proportionality. Since only the relative behavior of the integral length scale is of interest, the proportionality constant in the equilibrium turbulence considerations is take to be one, i.e. $L_I = L_\varepsilon$. For non-equilibrium turbulence, i.e. for the computations using the ε-correction, the Taylor length scales are multiplied with a factor K, which is determined such that $L_I = KL_\lambda = L_\varepsilon$ at (or near) the top dead center (TDC). The TDC, in fact the start of injection (SOI), is chosen because the non-equilibrium turbulence effects are mini-

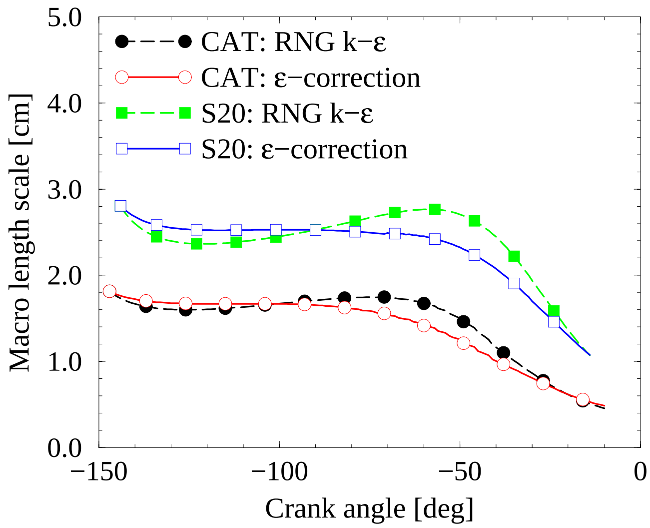

Figure 3: Turbulence macro length scales during the compression phase for the CAT and the S20 engines.

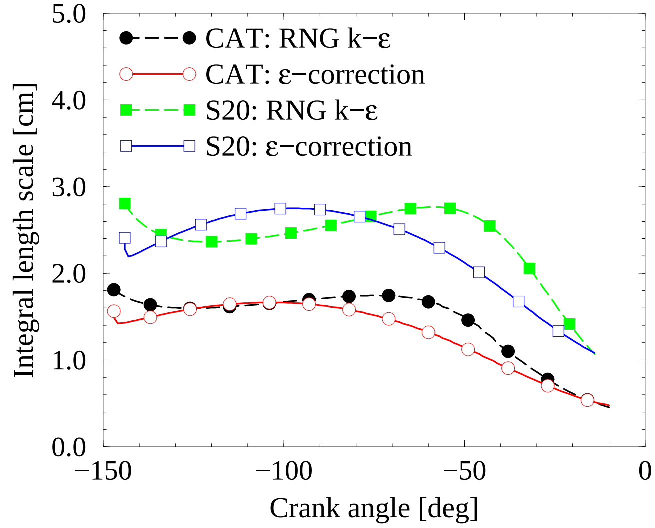

Figure 4: Turbulence integral length scales during the compression phase for the CAT and the S20 engines.

mal due to the low piston speed. For the simulations of the CAT and the S20 engines, the non-equilibrium proportionality constants were found to satisfy $K_{CAT} = 25$ and $K_{S20} = 45$.

The integral length scale from the experimental data has been determined from the *Lombardini* engine by matching the computed Taylor length scale to the experimental integral length scale at the beginning of the computations. This has resulted in a proportionality factor of 6.0.

Model Validation

The ε-correction model has been validated during the compression and expansion phase in a modified single-cylinder *Lombardini* engine operating in motored mode. Particular attention has been given to the behavior of the integral and macro length scales, L_I and L_ε, respectively. In the compression phase, the mean gas velocity is induced by the piston movement, which results in the formation of a squish flow whose intensity is increasing with decreasing cylinder volume.

The behavior of the length scales at the observation point A (cf. Fig. 1) are presented in Fig. 2. The integral length scale for the ε-correction has been computed as described in the previous subsection and reflects the experimental data very well. This behavior is consistent with the fact that the size of the largest eddies, which are determined by the geometry of the flow, start to decrease in the late compression phase due to the decrease in the cylinder volume. There is no apparent reason for the largest eddy sizes to increase in the compression phase, and therefore, the integral length scale predicted with the ε-correction is more realistic than the one obtained by means of the *RNG* $k-\varepsilon$ model, where $L_I \propto L_\varepsilon$.

The macro length scales, obtained with and without the ε-correction, are also shown in Fig. 2. Note that for non-equilibrium turbulence, an increase in L_ε during the compression phase can be explained by Eq. (4) in terms of the increase of the turbulence Reynolds number in the compression phase. In contrast, L_ε computed for equilibrium turbulence is assumed to be proportional to L_I, and since the latter is decreasing in the compression phase (L_I is determined by the geometry of the flow), the macro length scale would have to decrease also in this case.

Compression Computations

The compression phase for the CAT and the S20 engines has been simulated from the inlet valve closure (IVC) to the beginning of the fuel injection. For both engines, the initial turbulent kinetic energy at IVC has been set uniformly to 1.6 times the average kinetic piston energy.

At IVC the pistons are accelerating until they reach their maximum speed at around -90 deg from top dead center (TDC), after which they start to decelerate until they reach zero at TDC. In the compression phase, the mean gas velocity is induced by the piston movement, which results in the formation of a squish flow whose intensity is increasing with decreasing cylinder volume. In addition, the gas compressibility during the compression phase, given by $div\mathbf{u} < 0$, where $\mathbf{u}$ is the mean gas velocity, leads to increased shear stresses and thus contributes to the production and dissipation of turbulence, a fact which is readily seen from the k-ε transport equations [1]. The (mass-) averaged turbulence length scales for the standard *RNG* $k-\varepsilon$ model and the ε-correction are shown in Figs. 3 and 4 for both, the CAT and the S20 engines.

The influence of the dissipation correction on the macro length scales is shown in Fig. 3. In contrast to the *RNG* $k-\varepsilon$ model, the ε-correction macro length scales assume a constant level soon after IVC and remain constant until roughly -90 deg TDC after which they start to decrease. Because $L_I \propto L_\varepsilon$ for the *RNG* $k-\varepsilon$ model computations, the continued increase of the L_ε in the second halve of the compression phase is unrealistic, as has already been pointed out for the *Lombardini* engine.

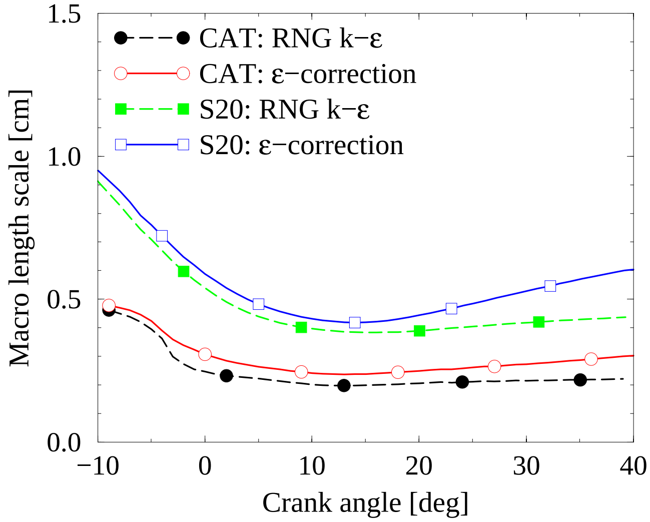

Figure 5: Turbulence macro length scales during the combustion phase for the CAT and the S20 engines.

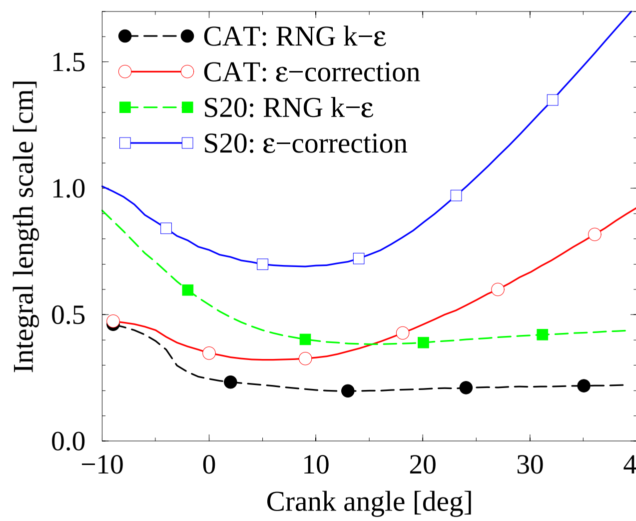

Figure 6: Turbulence integral length scales during the combustion phase for the CAT and the S20 engines.

Also, it can be seen in Fig. 3 that the macro length scales, with and without the ε-correction, are almost identical at the end of the compression phase. This is not surprising if one keeps in mind that the piston speed at this point is almost zero and therefore, the non-equilibrium turbulence effects are minimal.

The integral length scales for the compression phase are shown in Fig. 4 for both engines. As is described in the previous subsection, the integral length scale of the simulations can only be determined up to a constant of proportionality. The constant for the equilibrium turbulence has been set to one, whereas for the non-equilibrium turbulence, these constants have been found to be 25 and 45 for the CAT and the S20, respectively. The integral length scale predictions, using the ε-correction, is consistent with the fact that the size of the largest eddies, which are determined by the geometry of the squish flow, start to decrease around -90 CA TDC due to the decrease of the cylinder volume. The *RNG* $k-\varepsilon$ model computations, however, shows a continued increase of the integral length scales during the compression phase way beyond -90 CA TDC, and start their decrease considerably after their respective ε-correction length scales. As discussed in the case of the *Lombardini* engine, this is in contradiction to the fact that the largest eddies, and hence the L_I, are limited by the decreasing cylinder volume.

It can be concluded that the predictions of the compression turbulence is improved with the ε-correction, especially, the predictions of the macro length scales look much more realistic and are consistent with theoretical considerations.

Combustion Computations

The turbulence length scales of the CAT and the S20 are shown in Fig. 5 for the combustion phase, with and without the dissipation correction. For both engines, the ε-correction yields slightly higher values than the *RNG* $k-\varepsilon$ computations. As is discussed in the compression case, the values computed by means of the two different methods agree at the beginning of injection because the non-equilibrium turbulence effects are minimal at this point. Further, with continued expansion of the piston, the difference between equilibrium and non-equilibrium turbulence is amplified.

The integral length scales have been determined as discussed in the previous subsection and are shown in Fig. 6. The difference between equilibrium and non-equilibrium turbulence, i.e. between the *RNG* k–ε and the ε-correction, respectively, are considerable. Since the piston is expanding rapidly in the late compression phase, the cylinder volume is increasing and the integral length scale is increasing accordingly. This behavior is well reflected by the ε-correction integral length scales. In contrast, the *RNG* $k-\varepsilon$ computations predict an almost constant integral length scale during this expansion phase, which is inconsistent with the increasing cylinder volume determining the squish flow.

Summary and Conclusions

A correction for the turbulence dissipation rate, based on non-equilibrium turbulence considerations from rapid distortion theory, has been derived and implemented in combination with the *RNG* $k-\varepsilon$ turbulence model in a *KIVA*-based code. In particular, the turbulence dissipation rate has been adjusted via the turbulence Reynolds number by the relation

$$\varepsilon \propto Re_t^{-1} \varepsilon_{eq}$$

This model correction has been validated with experimental data in the compression and expansion phase of a small, high-compression engine in motored mode. The dissipation correction resulted in improved predictions of the length scales when compared with the *RNG* $k-\varepsilon$ model.

This model correction has been tested in the compression and combustion phase of two heavy-duty DI diesel engines. The turbulence behavior of the ε-correction shows clear improvements over the standard $RNG\ k-\varepsilon$ model computations. In particular, the integral and the macro length scales show improved behavior over the entire compression and combustion phase and are consistent with theoretical arguments.

Preliminary investigations which focus on the influence of the ε-correction on the pollution prediction of the two engines show encouraging results, and are in agreement with experimentally observed data.

Acknowledgment

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